Conditional Probability

CCM2 Unit 6: Probability

Conditional Probability

- **Conditional Probability:** A probability where a certain prerequisite condition has already been met.

- For example:
  - What is the probability of selecting a queen given an ace has been drawn and not replaced.
  - What is the probability that a student in the 10th grade is enrolled in biology given that the student is enrolled in CCM2?
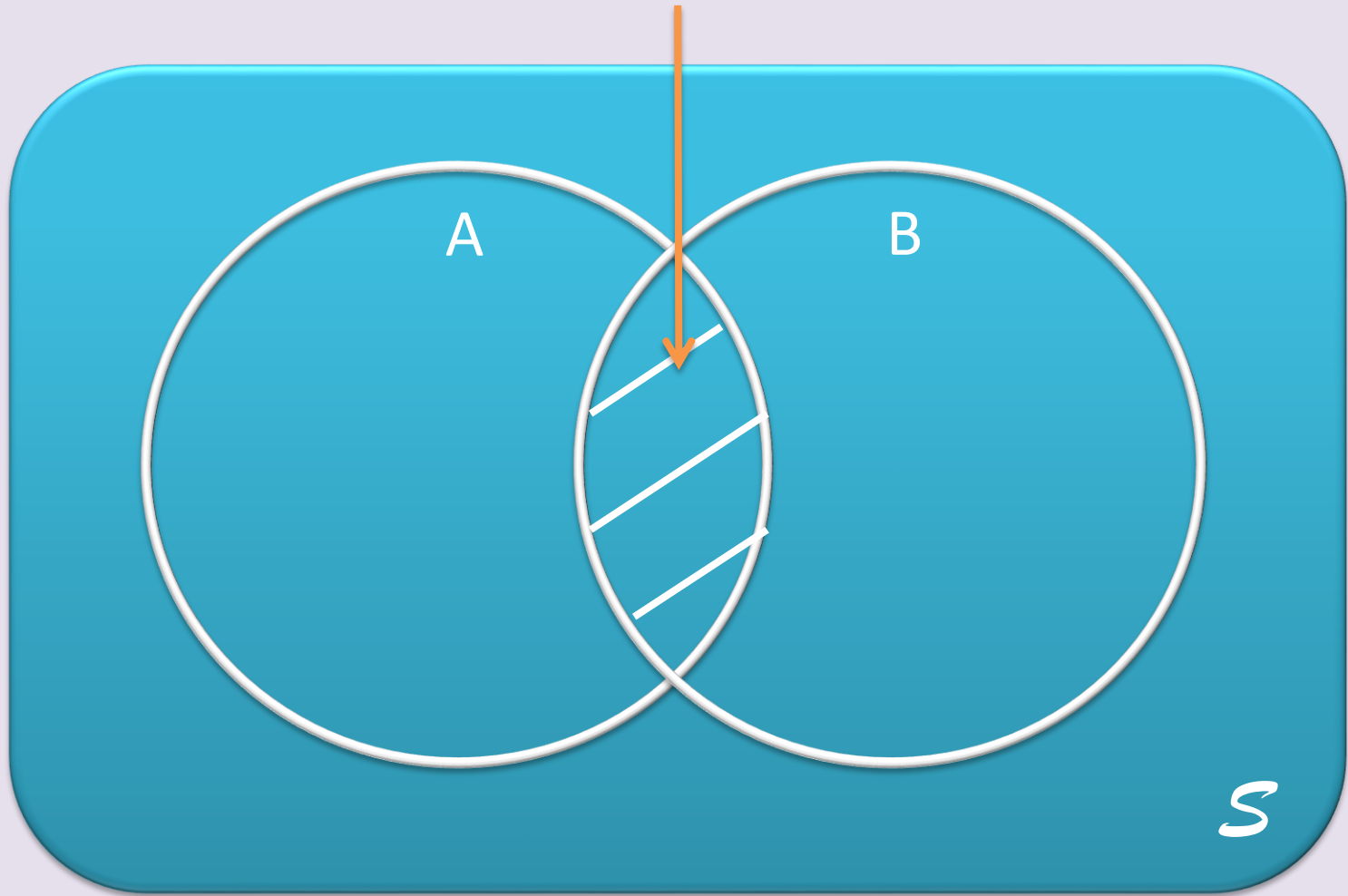
- Video about Conditional Probability

Conditional Probability Formula

- The conditional probability of A given B is expressed as P(A | B)

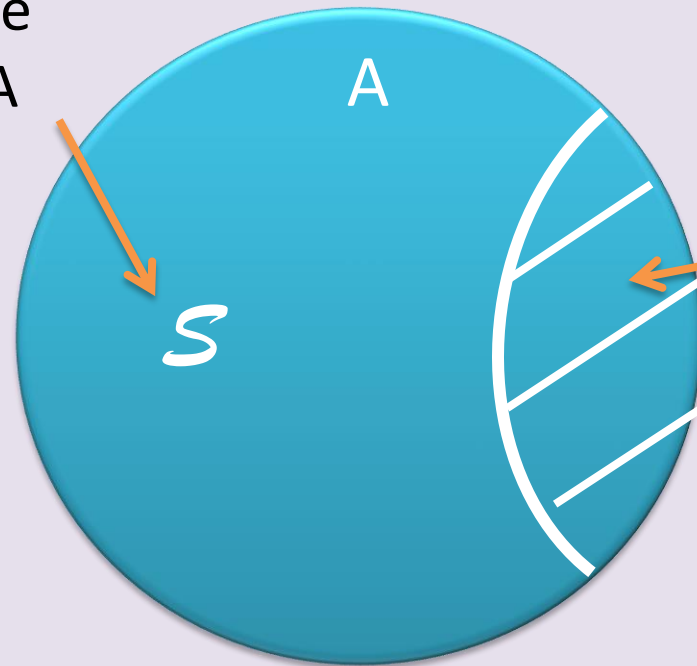$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

Joint Probability

P(A and B)

Conditional Probability

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

Since Event A has happened, the sample space is reduced to the outcomes in A

A

s

P(A and B) represents the outcomes from B that are included in A

Examples

1. You are playing a game of cards where the winner is determined by drawing two cards of the same suit. What is the probability of drawing clubs on the second draw if the first card drawn is a club?

P(club|club)

= P(2nd club and 1st club)/P(1st club)

= (13/52 x 12/51)/(13/52)

= 12/51 or 4/17

The probability of drawing a club on the second draw given the first card is a club is 4/17 or 23.5%

2. A bag contains 6 blue marbles and 2 brown marbles. One marble is randomly drawn and discarded. Then a second marble is drawn. Find the probability that the second marble is brown given that the first marble drawn was blue.

P(brown|blue)

= P(brown and blue)/P(blue)

= (6/8 x 2/7)/(6/8)

= 2/7

The probability of drawing a brown marble given the first marble was blue is 2/7 or 28.6%

3. In Mr. Jonas' homeroom, 70% of the students have brown hair, 25% have brown eyes, and 5% have both brown hair and brown eyes. A student is excused early to go to a doctor's appointment. If the student has brown hair, what is the probability that the student also has brown eyes?

P(brown eyes|brown hair)

= P(brown eyes and brown hair)/P(brown hair)

= .05/.7

= .071

The probability of a student having brown eyes given he or she has brown hair is 7.1%

Using Two-Way Frequency Tables to Compute Conditional Probabilities

- In CCM1 you learned how to put data in a two-way frequency table (using counts) or a two-way relative frequency table (using percents), and use the tables to find joint and marginal frequencies and conditional probabilities.

- Let's look at some examples to review this.

1. Suppose we survey all the students at school and ask them how they get to school and also what grade they are in. The chart below gives the results. Complete the two-way frequency table:

	Bus	Walk	Car	Other	Total
9th or 10th	106	30	70	4	
11th or 12th	41	58	184	7	
Total					

	Bus	Walk	Car	Other	Total
9th or 10th	106	30	70	4	210
11th or 12th	41	58	184	7	290
Total	147	88	254	11	500

Suppose we randomly select one student.

a. What is the probability that the student walked to school?
- 88/500
- 17.6%

b. P(9th or 10th grader)
- 210/500
- 42%

c. P(rode the bus OR 11th or 12th grader)
- 147/500 + 290/500 – 41/500
- 396/500 or 79.2%

	Bus	Walk	Car	Other	Total
9th or 10th	106	30	70	4	210
11th or 12th	41	58	184	7	290
Total	147	88	254	11	500

d. What is the probability that a student is in 11th or 12th grade *given that* they rode in a car to school?

P(11th or 12th|car)

* We only want to look at the car column for this probability!

= 11th or 12th graders in cars/total in cars

= 184/254 or 72.4%

The probability that a person is in 11th or 12th grade given that they rode in a car is 72.4%

	Bus	Walk	Car	Other	Total
9th or 10th	106	30	70	4	210
11th or 12th	41	58	184	7	290
Total	147	88	254	11	500

e. What is P(Walk|9th or 10th grade)?

= walkers who are 9th or 10th / all 9th or 10th

= 30/210

= 1/7 or 14.2%

The probability that a person walks to school given he or she is in 9th or 10th grade is 14.2%

2. The manager of an ice cream shop is curious as to which customers are buying certain flavors of ice cream. He decides to track whether the customer is an adult or a child and whether they order vanilla ice cream or chocolate ice cream. He finds that of his 224 customers in one week that 146 ordered chocolate. He also finds that 52 of his 93 adult customers ordered vanilla. Build a two-way frequency table that tracks the type of customer and type of ice cream.

	Vanilla	Chocolate	Total
Adult			
Child			
Total			

	Vanilla	Chocolate	Total
Adult	52		93
Child			
Total		146	224

	Vanilla	Chocolate	Total
Adult	52	41	93
Child	26	105	131
Total	78	146	224

a. Find P(vanilla|adult)

= 52/93

= 55.9%

b. Find P(child|chocolate)

= 105/146

=71.9%

3. A survey asked students which types of music they listen to? Out of 200 students, 75 indicated pop music and 45 indicated country music with 22 of these students indicating they listened to both. Use a Venn diagram to find the probability that a randomly selected student listens to pop music given that they listen country music.

Pop

Country

22

23

53

102
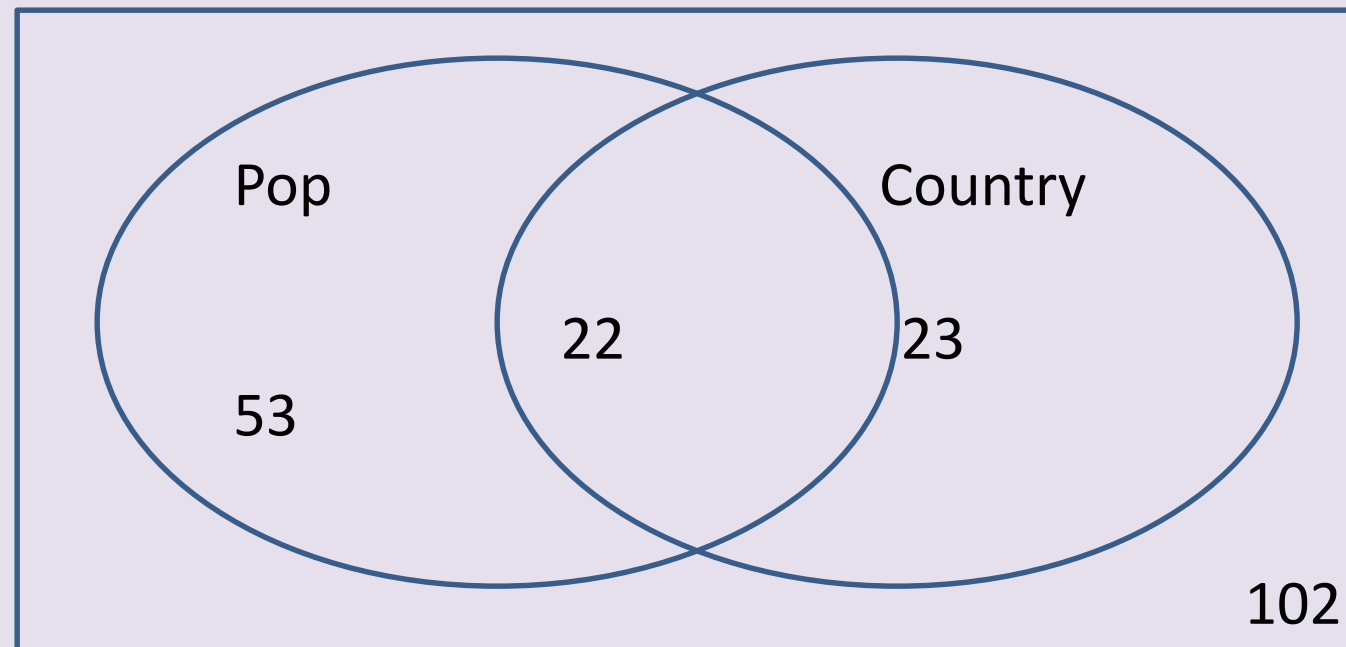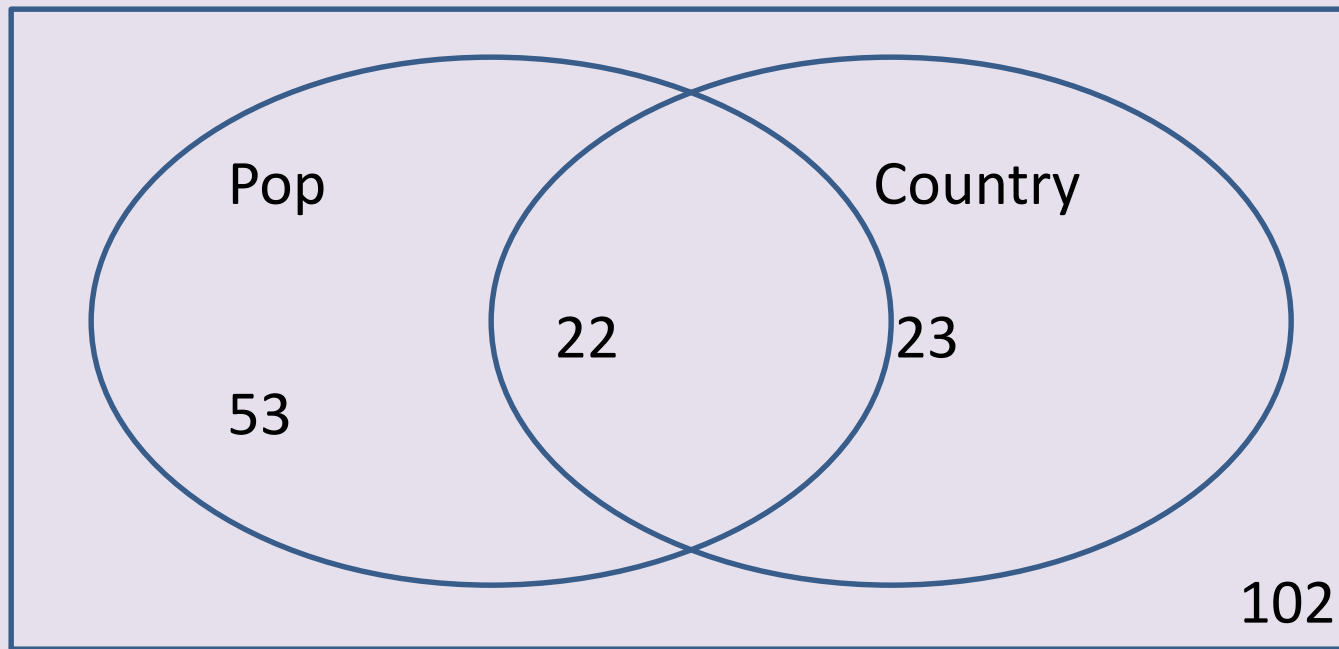
P(Pop|Country)

= 22/(22+23)

= 22/45 or 48.9%

48.9% of students who listen to country also listen to pop.

Using Conditional Probability to Determine if Events are Independent

- If two events are statistically independent of each other, then:

$$P(A|B) = P(A) \text{ and } P(B|A) = P(B)$$

- Let's revisit some previous examples and decide if the events are independent.

1. You are playing a game of cards where the winner is determined by drawing two cards of the same suit. Each player draws two cards, without replacement. What is the probability of drawing clubs on the second draw if the first card drawn is a club? Are the two events independent?

Let event A = draw a club and event B = draw a club.

$$P(A) = \frac{13}{52} \text{ or } \frac{1}{4} = .25$$

13 clubs out of 52 cards

$$P(B) = \frac{13}{52} \text{ or } \frac{1}{4} = .25$$

P(drawing a second club after drawing the first club) =

$$P(B|A) = \frac{12}{51} \text{ or } \frac{4}{17} \approx .235$$

Only 12 clubs left and only 51 cards left

So, $P(B) \neq P(B|A)$

Thus, the events of drawing a club followed by drawing another club without replacement are NOT independent events.

3. In Mr. Jonas' homeroom, 70% of the students have brown hair, 25% have green eyes, and 5% have both brown hair and green eyes. A student is excused early to go to a doctor's appointment. If the student has brown hair, what is the probability that the student also has green eyes? Let A = brown hair and B = green eyes. Are events A and B independent?
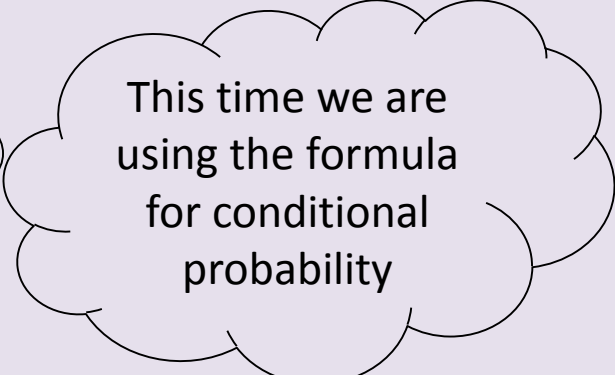
P(A) = P(brown hair) = .7

P(B) = P(green eyes) = .25

P(brown hair and green eyes) P(A and B) = .05

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{.05}{.25} = .20$$

This time we are using the formula for conditional probability

P(A) ≠ P(A|B)

Thus, the events are dependent!

	Vanilla	Chocolate	Total
Adult	52	41	93
Child	26	105	131
Total	78	146	224

4. Determine whether age and choice of ice cream are independent events.

We could start by looking at the P(vanilla|adult) and P(vanilla). If they are the same, then the events are independent.

P(vanilla|adult) = 52/93 = 55.9%

P(vanilla) = 78/224 = 34.8%

P(vanilla|adult) ≠ P(vanilla), so the events are dependent!