Geometric Proofs Involving Complementary and Supplementary Angles

October 18, 2010

SWBAT: Recognize complementary and supplementary angles and prove angles congruent by means of four new theorems.

Warm - Up

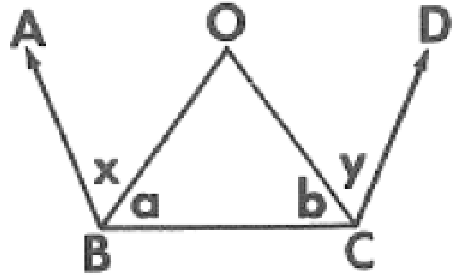
Given:

$m\angle ABC = m\angle DCB.$

$m\angle a = m\angle b.$

Prove:

$m\angle x = m\angle y.$

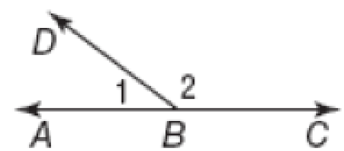
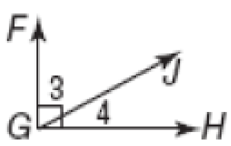


	Statements		Reasons
1	$m\angle ABC = m\angle DCB$	1	given
2	$m\angle a = m\angle b$	2	given
3	$m\angle ABC - m\angle a = m\angle DCB - m\angle b$	3	Subtraction prop (1,2)
4	$m\angle x = m\angle y$	4	Substitution Prop
5		5	
6		6	
7		7	
8		8	

SWBAT: Recognize complementary and supplementary angles and prove angles congruent by means of four new theorems.

Proving Angle Relationships

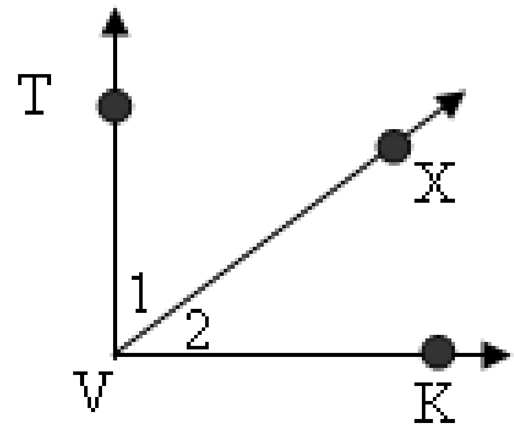
The two postulates can be used to prove the following two theorems.

Supplement Theorem	If two angles form a linear pair, then they are supplementary angles. **Example:** If $\angle 1$ and $\angle 2$ form a linear pair, then $m\angle 1 + m\angle 2 = 180$.	
Complement Theorem	If the noncommon sides of two adjacent angles form a right angle, then the angles are complementary angles. **Example:** If $\overleftrightarrow{GF} \perp \overleftrightarrow{GH}$, then $m\angle 3 + m\angle 4 = 90$.	

SWBAT: **Recognize complementary and supplementary angles and prove angles congruent by means of four new theorems.**

Given: ∠TVK is a right angle.

Prove: ∠1 is complementary to ∠2.



SWBAT: Recognize complementary and supplementary angles and prove angles congruent by means of four new theorems

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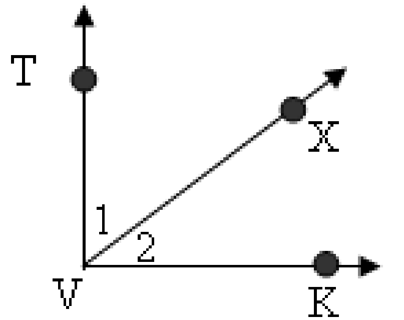
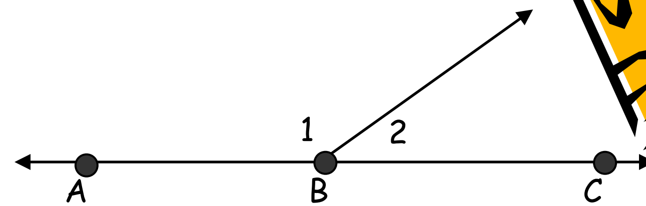


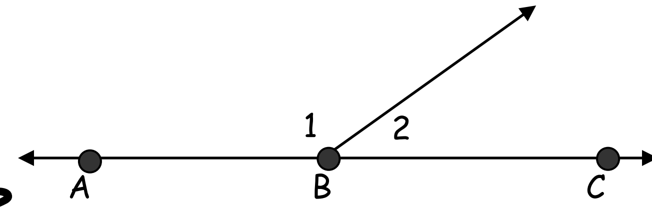
Statements	Reasons
1. ∠TVK is a right angle.	1. Given
2. m ∠TVK = 90°	2. A right angle measures 90 °
3. ∠1 + ∠2 = ∠TVK	3. Angle Addition Postulate
4. ∠1 + ∠2 = 90°	4. Substitution
5. ∠1 is complementary to ∠2.	5. Definition of Complementary ∠s

SWBAT: Recognize complementary and supplementary angles and prove angles congruent by means of four new theorems.

Given: $\angle ABC$ is a straight angle
Prove: $\angle 1$ is supplementary to $\angle 2$.



SWBAT: Recognize complementary and supplementary angles and prove angles congruent by means of four new theorems.



Given: $\angle ABC$ is a straight angle

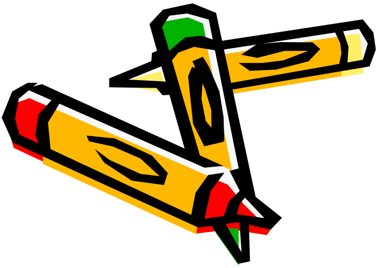
Prove: $\angle 1$ is supplementary to $\angle 2$.

Statements	Reasons
1. $\angle ABC$ is a straight angle	1. Given
2. $\angle 1 + \angle 2 = \angle ABC$	2. Angle Addition Postulate
3. $\angle 1$ and $\angle 2$ form a Linear pair	3. Definition of Linear Pairs
4. $\angle 1$ is supplementary to $\angle 2$	4. Linear Pairs form supplementary angles.

<u>SWBAT:</u> Recognize complementary and supplementary angles and prove angles congruent by means of four new theorems.

Congruent and Right Angles The Reflexive Property of Congruence, Symmetric Property of Congruence, and Transitive Property of Congruence all hold true for angles. The following theorems also hold true for angles.

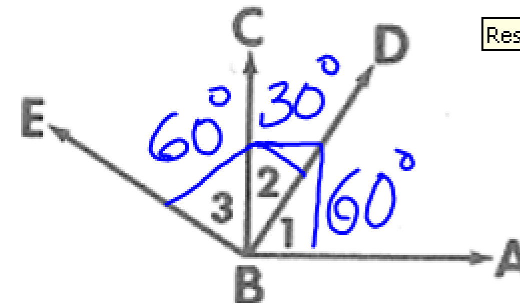
Congruent Supplements Theorem	Angles supplement to the same angle or congruent angles are congruent.
Congruent Complement Theorem	Angles complement to the same angle or to congruent angles are congruent.
Vertical Angles Theorem	If two angles are vertical angles, then they are congruent.
Perpendicular Lines Theorem	Perpendicular lines intersect to form four right angles.
Right Angles Theorem	All right angles are congruent.
Theorem #1	Perpendicular lines form congruent adjacent angles.
Theorem #2	If two angles are congruent and supplementary, then each angle is a right angle.
Theorem #3	If two congruent angles form a linear pair, then they are right angles.



SWBAT: Recognize complementary and supplementary angles and prove angles congruent by means of four new theorems.

3. *Given:* ∠1 is complementary to ∠2.
∠3 is complementary to ∠2.

Prove: ∠1 ≅ ∠3.



Statements	Reasons
1 ∠1 is compl. to ∠2	1 given
2 ∠3 is compl. to ∠2	2 given
3 m∠1 + m∠2 = 90	3 def. of compl.
4 m∠3 + m∠2 = 90	4 def. of compl.
5 m∠1 + m∠2 = m∠3 + m∠2	5 Substitution (3,4)
6 m∠1 = m∠3	6 Subtraction prop.
7 ∠1 ≅ ∠3	7 def. of ≅ ∠'s
8	8

SWBAT: Recognize complementary and supplementary angles and prove angles congruent by means of four new theorems.

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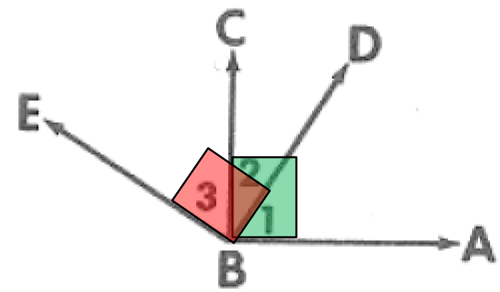
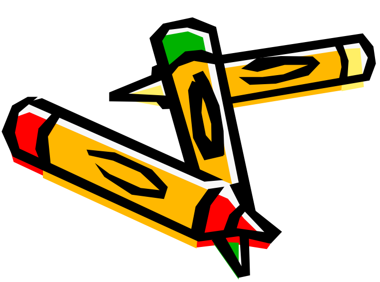


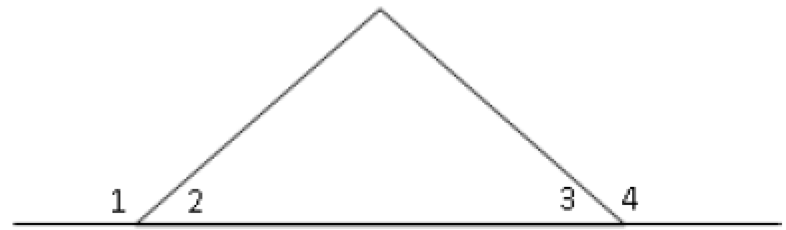
Statements	Reasons
1. ∠1 is complementary to ∠2	1. given
∠3 is complementary to ∠2	
2. ∠1 ≅ ∠3	2. ≅ Complements Theorem



<u>SWBAT:</u> Recognize complementary and supplementary angles and prove angles congruent by means of four new theorems.

4. Given: $\angle 1 \cong \angle 4$

   Prove: $\angle 2 \cong \angle 3$

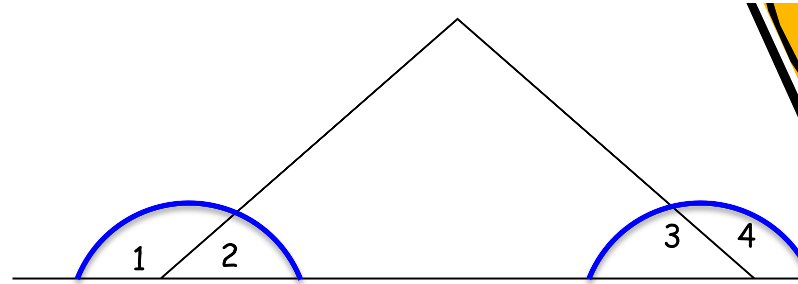


Statements	Reasons
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8

SWBAT: Recognize complementary and supplementary angles and prove angles congruent by means of four new theorems.

Given: $\angle 1 \cong \angle 4$

Prove: $\angle 2 \cong \angle 3$

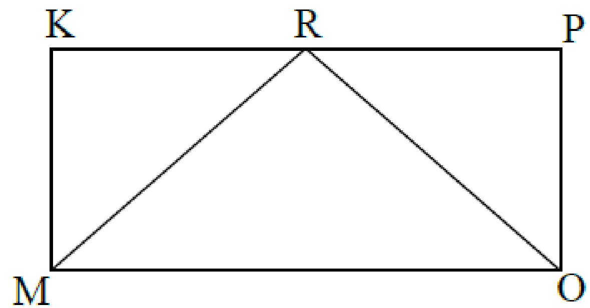


Statements	Reasons
1. $\angle 1 \cong \angle 4$	1. given
2. $\angle 1$ and $\angle 2$ form a Linear pair $\angle 3$ and $\angle 4$ form a Linear pair	2. Definition of Linear Pair
3. $\angle 1$ is suppl. to $\angle 2$ $\angle 3$ is suppl. to $\angle 4$	3. Linear Pairs form Supplementary angles.
4. $\angle 2 \cong \angle 3$	4. $\cong$ Supplements Theorem

SWBAT: Recognize complementary and supplementary angles and prove angles congruent by means of four new theorems.

5. Given: $\overline{KM} \perp \overline{MO}$
$\overline{PO} \perp \overline{MO}$
$\angle KMR \cong \angle POR$

Prove: $\angle ROM \cong \angle RMO$



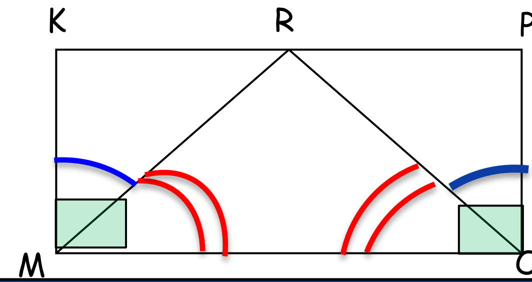
Statements	Reasons
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8

5. Given: $\overline{KM} \perp \overline{MO}$

$\overline{PO} \perp \overline{MO}$

∠KMR ≅ ∠POR

Prove: ∠ROM ≅ ∠RMO



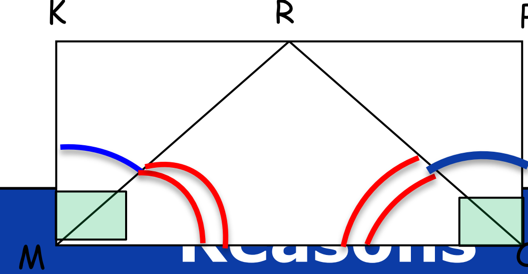
Statements	Reasons
1. $\overline{KM} \perp \overline{MO}$ $\overline{PO} \perp \overline{MO}$ ∠KMR ≅ ∠POR	1. given
2. ∠KMO and ∠POM are right angles	2. Definition of ⊥ lines
3. ∠KMO ≅ ∠POM	3. All right Angles are ≅
4. ∠KMO - ∠KMR ≅ ∠POM - ∠RMO	4. Subtraction Property (3, 1)
5. ∠RMO ≅ ∠ROM	5. Substitution Property
6. ∠ROM ≅ ∠RMO	6. Symmetric Property

5. Given: $\overline{KM} \perp \overline{MO}$

$\overline{PO} \perp \overline{MO}$

$\angle KMR \cong \angle POR$

Prove: $\angle ROM \cong \angle RMO$



Statements	Reasons
1. $\overline{KM} \perp \overline{MO}$ $\overline{PO} \perp \overline{MO}$ $\angle KMR \cong \angle POR$	1. given
2. $\angle KMO$ is a right angle $\angle POM$ is a right angle	2. Definition of $\perp$ lines
3. $\angle KMO \cong 90°$ $\angle POM \cong 90°$	3. Right $\angle$s are $\cong$ to 90°
4. $\angle KMR + \angle RMO \cong \angle KMO$ $\angle POR + \angle ROM \cong \angle POM$	4. Angle Addition Postulate
5. $\angle KMR + \angle RMO \cong 90$ $\angle POR + \angle ROM \cong 90$	5. Substitution (3 into 4)
5. $\angle KMR$ is compl. to $\angle RMO$ $\angle POR$ is compl. to $\angle ROM$	5. Definition of Complementary $\angle$s
6. $\angle ROM \cong \angle RMO$	6. $\cong$ Complements Theorem